GCSE Maths – Number

Terminating and Recurring Decimals

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of terminating and recurring decimals questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Work out $0.55 + \frac{1}{4}$

Step 1: Convert both terms to the same form.

For this question, we will convert them into terminating decimals.

To convert $\frac{1}{4}$ to a decimal, we divide 1 by 4. This gives us 0.25. This is a terminating decimal because we can see the end of the numbers.

Step 2: Perform the operation once both are in the same form.

You can use any preferred method of addition, for example, column addition, to perform the calculation.

$$0.55 + 0.25 = \mathbf{0.80}$$

Guided Example

Work out $0.9 - \frac{2}{5}$

Step 1: Convert both terms to the same form.

For this question, we will convert both terms into terminating decimals.

$\frac{2}{5} = 0.4$

Divide 2 by 5, this gives 0.4 which is a terminating decimal

Step 2: Perform the operation once both are in the same form.

$0.9 - 0.4 = 0.5$

Perform the operation, perhaps using column subtraction.

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

1. Work out the following, writing the final answer as a terminating decimal.

a) $0.75 + \frac{1}{5}$

$\frac{1}{5} = 0.2 \rightarrow 0.75 + 0.2 = 0.75 + 0.20 = 0.95$

Divide 1 by 5. This gives 0.2 (A terminating decimal).

Perform the operation, using column addition.

b) $2.4 + \frac{3}{4}$

$\frac{3}{4} = 0.75 \rightarrow 2.4 + 0.75 = 2.40 + 0.75 = 3.15$

Divide 3 by 4. This gives 0.75 (A terminating decimal).

Perform the operation, using column addition.

c) $\frac{4}{5} - 0.1$

$\frac{4}{5} = 0.8 \rightarrow 0.8 - 0.1 = 0.8 - 0.1 = 0.7$

Divide 4 by 5. This gives 0.8 (A terminating decimal).

Perform the operation, using column subtraction.

d) $\frac{1}{10} + 0.999$

$\frac{1}{10} = 0.1 \rightarrow 0.1 + 0.999 = 0.100 + 0.999 = 1.099$

Divide 1 by 10. This gives 0.1 (A terminating decimal).

Perform the operation, using column addition.

Section B

Worked Example

Calculate $0.5 + 0.3$. Write the answer as a fraction.

Step 1: If necessary, convert both terms to terminating decimals. Then perform the operation.

We do not need to convert these terms because they are already in terminating decimal form.

$$0.5 + 0.3 = 0.8$$

Step 2: To convert the decimal result to a fraction, identify the numbers in the unit, tenths and hundredths column (and more if appropriate).

For this number, we have 8 tenths only. This can be written as $\frac{8}{10}$.

Step 3: Simplify the fraction if possible.

$\frac{8}{10}$ can be simplified to $\frac{4}{5}$, by dividing the numerator and denominator by 2.

Hence, the final answer is $\frac{4}{5}$.

Guided Example

Calculate $0.8 - 0.3$. Write the answer as a fraction.

Step 1: If necessary, convert both terms to terminating decimals. Then perform the operation.

$0.8 - 0.3 = 0.5$

Step 2: To convert the decimal to a fraction, identify the numbers in the unit, tenths and hundredths column (and more if appropriate).

units tenths
$0.5 = \frac{5}{10}$

Step 3: Simplify the fraction if possible.

$\frac{5}{10} = \frac{5 \times 1}{5 \times 2} = \frac{1}{2}$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

2. Calculate the following, writing the answer as a fraction.

a) $0.25 + \frac{1}{4}$

$\frac{1}{4} = 0.25 \rightarrow 0.25 + 0.25 = 0.25 + 0.25 = 0.50 = \frac{5}{10} = \frac{5 \times 1}{5 \times 2} = \frac{1}{2}$

Divide 1 by 4. This gives 0.25 (A terminating decimal).

Perform the operation, using column addition.

Convert decimal to a fraction and simplify.

b) $0.3 + 0.6$

$0.3 + 0.6 = 0.3 + 0.6 = 0.9 = \frac{9}{10}$

Perform the operation, using column addition.

Convert decimal to a fraction and simplify.

c) $\frac{2}{5} - \frac{1}{10}$

$\frac{2}{5} = \frac{2}{5} \times \frac{2}{2} = \frac{4}{10} \rightarrow \frac{4}{10} - \frac{1}{10} = \frac{4-1}{10} = \frac{3}{10}$

Convert 'fifths' into 'tenths'

Perform the operation.

d) $2\frac{1}{2} + 0.4$

Perform the operation.

$2\frac{1}{2} = 2 + \frac{1}{2} = 2 + 0.5 = 2.5 \rightarrow 2.5 + 0.4 = 2.9 = \frac{29}{10}$

Convert Fraction to a decimal and simplify.

Convert decimal to a fraction and simplify.

Section C – Higher Only

Worked Example

Convert $\frac{2}{3}$ to a recurring decimal

Step 1: To obtain the decimal, we simply divide the numerator by the denominator.

Using the 'bus-stop' method or another method, we find $2 \div 3 = 0.6666\ldots$

Step 2: Write the decimal in its recurring form.

Identify the numbers that recur. For this number, it is just 6. We place a dot above 6 as it is the only number that is recurring.

$$0.66666\ldots = 0.\dot{6}$$

Guided Example

Write $\frac{5}{6}$ as a recurring decimal

Step 1: To obtain the decimal, we simply divide the numerator by the denominator. Use the bus stop method to help with the division.

$\frac{5}{6} =$ 6 ⟌ 5.0²0²0²0 = 0.8333

Step 2: Write the decimal in its recurring form.

$0.8333 = 0.8\dot{3}$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

3. Write the following fractions as recurring decimals:

a) $\frac{7}{9}$

$9\overline{)7.0\,0\,0} = 0.777\ldots = 0.\dot{7}$

Use bus stop method to divide

Rewrite recurring decimal with the recurring symbol

b) $\frac{4}{3}$

$3\overline{)4.0\,0\,0} = 1.333\ldots = 1.\dot{3}$

Use bus stop method to divide

Rewrite recurring decimal with the recurring symbol

c) $\frac{13}{6}$

$6\overline{)13.0\,0\,0} = 2.166\ldots = 2.1\dot{6}$

Use bus stop method to divide

Rewrite recurring decimal with the recurring symbol

d) $\frac{1}{13}$

$13\overline{)1.0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0} = 0.076923076923 = 0.\dot{0}76923$

Use bus stop method to divide

Rewrite recurring decimal with the recurring symbol

Section D – Higher Only

Worked Example

Convert $0.\dot{4}\dot{5}$ to a fraction

Step 1: Write out an equation to show that an unknown fraction, *x*, is equal to the recurring decimal. Write the recurring decimal out to show several numbers.

$$x = 0.45454545\ldots$$

Step 2: Multiply both sides of the equation by a number that gives the same recurring numbers after the decimal.

To obtain the same recurring numbers after the decimal place (if we think of the number as $0.4545454545\ldots$*) then we must multiply by* 100*.*

$$100x = 45.45454545\ldots$$

Step 3: Subtract the first equation from the second equation.

$$100x - x = 45.45454545\ldots - 0.45454545\ldots$$

$$99x = 45$$

Step 4: Rearrange the equation to solve for x.

$$99x = 45$$

$$x = \frac{\mathbf{45}}{\mathbf{99}}$$

Guided Example

Write $0.\dot{1}1\dot{9}$ as a fraction

Step 1: Write out an equation to show that an unknown fraction, x, is equal to the recurring decimal. Write the recurring decimal out to show several numbers.

$x = 0.\dot{1}1\dot{9}$

Step 2: Multiply both sides of the equation by a number that gives the same recurring numbers after the decimal. This gives a second equation.

$x = 0.\dot{1}1\dot{9}$

$1000x = 119.\dot{1}1\dot{9}$

Step 3: Subtract the first equation from the second equation and rearrange to solve for x.

$1000x - x = 119.\dot{1}1\dot{9} - 0.\dot{1}1\dot{9} = 119$

$999x = 119$

$x = \frac{119}{999}$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

4. Write the following recurring decimals as fractions.

a) $0.\dot{2}$

① $x = 0.222...$

② $10x = 2.222...$

Set decimal equal to x and multiply until numbers repeat again.

→ ② − ① = $10x - x = 2.222 - 0.222 = 2$

$9x = 2$

$x = \frac{2}{9}$

subtract the two equations from each other and rearrange for x.

b) $0.\dot{9}2\dot{3}$

① $x = 0.923923...$

② $1000x = 923.923...$

Set decimal equal to x and multiply until numbers repeat again.

→ ② − ① $1000x - x = 923.923 - 0.923$

$999x = 923$

$x = \frac{923}{999}$

subtract the two equations from each other and rearrange for x.

c) $0.\dot{4}6291\dot{}$

① $x = 0.46291$

② $100,000x = 46291.46291$

Set decimal equal to x and multiply until numbers repeat again.

→ ② − ① $100,000x - x = 46291.46291 - 0.46291$

$99,999x = 46291$

$x = \frac{46,291}{99,999}$

subtract the two equations from each other and rearrange for x.

d) $1.\dot{4}$

① $x = 1.444...$

② $10x = 14.444...$

Set decimal equal to x and multiply until numbers repeat again.

→ ② − ① $10x - x = 14.44 - 1.44$

$9x = 13$

$x = \frac{13}{9}$

subtract the two equations from each other and rearrange for x.

e) $3.1\dot{6}$

① $x = 3.16$

② $100x = 316.16$

Set decimal equal to x and multiply until numbers repeat again.

→ ② − ① $100x - x = 316.16 - 3.16$

$99x = 313$

$x = \frac{313}{99}$

subtract the two equations from each other and rearrange for x.

f) $-29.5\dot{6}$

① $x = -29.5656...$

② $100x = -2956.5656...$

Set decimal equal to x and multiply until numbers repeat again.

→ ② − ① $100x - x = -2956.5656 - -29.56$

$99x = -2927$

$x = -\frac{2927}{99}$

subtract the two equations from each other and rearrange for x.